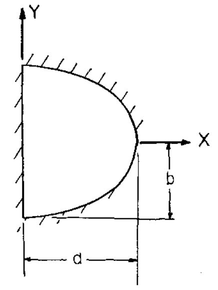

Fig. 1 Geometry of semi-elliptical plate

The motion of the plates (Figs. 1 and 2), assuming that the damping forces are proportional to the velocity, is governed by the following partial differential equation:

$$\nabla^4 w(x_1y_1t) + (k/D)w,_t(x_1y_1t) + (\rho h/D)w,_{tt}(x_1y_1t) = 0 \quad (1)$$

Choosing an ordinary product solution such as

$$w(x_1y_1t) = \phi(x_1y)e^{-\alpha t} \cos\omega t \quad (2)$$

and noting that Eq. (1) must hold for all time t, one obtains

$$\alpha = k/2\rho h \quad (3)$$

and

$$\nabla^4\phi(x_1y) - \lambda^2\phi(x_1y) = 0 \quad (4)$$

where

$$\lambda^2 = (k^2/4\rho hD)[1 + (2\rho h\omega/k)^2] \quad (5)$$

or

$$\omega = [(D\lambda^2/\rho h) - (k^2/4\rho^2h^2)]^{1/2} \quad (6)$$

Note that a typographical error in the value of ω appears in a previous paper by the author.[2] It is given correctly in Eq. (6).

Thus, if λ can be found, the natural frequency can be calculated. Let

$$\phi(x_1y) = \sum_{m=1}^{n} A_m\phi_m(x_1y) \quad (7)$$

where the ϕ_m are characteristic functions chosen so that they satisfy the geometrical boundary conditions of the problem. For a clamped plate, these conditions are that the deflection $w(\Gamma) = 0$ and the slope $w_{1n}(\Gamma) = 0$ on the boundary (Γ).

Using the Galerkin technique, it follows that

$$\iint_{\text{area}} L[\phi(x_1y)]\phi_i(x_1y)dxdy = 0 \quad (8)$$

where

$$L(\phi) = \nabla^4\phi - \lambda^2\phi \quad (9)$$

For the semi-elliptical plate, cutting the series off at two terms, the characteristic function ϕ is chosen to be

$$\phi(x_1y) = A_1\phi_1 + A_2\phi_2 = A_1x^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 + A_2x^4\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^3 \quad (10)$$

For the quarter-elliptical plate, cutting the series off at two terms, the characteristic function ϕ is chosen to be

$$\phi(x_1y) = B_1\phi_1 + B_2\phi_2 = B_1x^2y^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 + B_2x^4y^4\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)^2 \quad (11)$$

By substituting Eqs. (11) and (10) in Eq. (8) and setting the determinate of the coefficients of Eq. (8) equal to zero, one obtains for the semi-elliptical plate

$$\lambda^2 = (867/a^4)[1 + 0.271(a^2/b^2) + 0.111(a^4/b^4)] \quad (12)$$

For $a = b = 1$, $\lambda = 34.5$. For the quarter-elliptical plate, one obtains

$$\lambda^2 = (1105/a^4)\{(a^2/b^2) + .917[1 + (a^4/b^4)]\} \quad (13)$$

For $a = b = 1$, $\lambda = 56$. Some values of λ for different shape ratios are given in Table 1.

Table 1 λ for different shape ratios a/b and $a = 1$

a/b	Semi-ellipse λ	Quarter-ellipse λ
1.0	34.5	56
1.2	37.5	69
1.5	43.5	93
3.0	104.0	306
5.0	257.0	820
10.0	990.0	3200

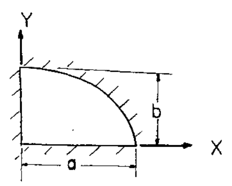

Fig. 2 Geometry of the quarter-elliptical plate

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[1] Stanisic, M. M., "Free vibration of a rectangular plate with damping considered," Quart. Appl. Math. **XII**, 361-367 (1955).

[2] McNitt, R. P., "Free vibration of a damped elliptical plate," J. Aerospace Sci. 29, 1124 (1962).

Development of a Stable "White" Coating System

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THE development of stable "white" coatings for space vehicle temperature control currently is receiving considerable attention in the aerospace industry. Recent optical reflection measurements performed on Corning #7941, multiform fused silica indicated that the bulk material had an unusually low solar absorptance of 0.08 and a high total hemispherical emittance of 0.77. The reason for this desirable combination of optical properties led to the development of a practical coating system with similar properties.

The multiform fused silica is essentially a conglomeration of fine particles of ultrapure, fused silica that has been sintered to form a free standing piece. The "whiteness" of the bulk material may be explained from the theory of optical scattering. The mixture of two optical media having different indices of refraction, with at least one having physical dimensions of the same order as the wavelength of light to be scattered, is the basic feature of a scattering layer. Corning #7941, multiform fused silica consists of a mixture of fine particles of fused silica (index of refraction = 1.46) and air (index of refraction = 1.0).

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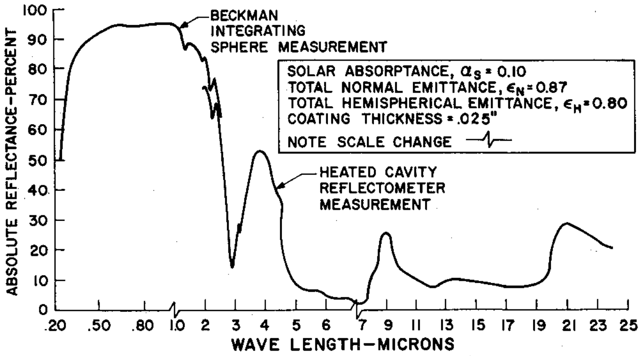

Fig. 1 Spectral reflectance of experimental coating

Table 1 Optimum coating design

Solar wavelength intervals, μ	Particle distribution, %	Particle size diam, μ
0.20 to 0.47	18	0.38
0.47 to 0.60	28	0.53
0.60 to 0.93	30	0.75
0.93 to 1.30	16	1.09
1.30 to 2.60	8	1.90

Under subcontract from the General Electric Company, Re-Entry Systems Department, Corning Glass Works in conjunction with Dow-Corning† formulated a paint whose optical properties approach those of the Corning #7941, multiform fused silica. The pigment consists of fine particles of ultrapure fused silica, and the paint vehicle is an experimental silicone varnish that evaporates at elevated temperature leaving a matrix of fine particles of fused silica with good adhesion. Figure 1 illustrates the results of optical reflection measurements performed on the experimental coating for a coating thickness of 25 mils. Reduction of the near-normal spectral reflection data yields a solar absorptance of 0.10 and a total hemispherical emittance of 0.80. The coating formulation presently is undergoing ultraviolet, vacuum exposure, and it is anticipated that the coating will be very stable since its singular ingredient, i.e., ultrapure fused silica, is transparent to the ultraviolet spectrum above 2000 Å. The Corning grade #7940 fused silica is also extremely resistant to particle radiation.

It is speculated that further optimization of the experimental coating (lower absorptance) may be achieved by adjusting the particle size and particle size distribution of the fused silica. The following equation defines the optimum particle size for maximum scattering of wavelength λ:[1]

$$d = (0.90\lambda/n_0\pi)[(m^2 + 2)/(m^2 - 1)]$$

where λ is the wavelength to be scattered in microns, n_0 is the index of refraction of the vehicle, m is the ratio of the index of refraction of the pigment of the vehicle, and d defines the optical particle size diameter in microns. The particle distribution would be determined by the spectrum of radiation to be scattered. Table 1 lists a possible particle size and particle size distribution for optimum reflection of solar illumination.

In addition to the spacecraft application, the coating has been suggested to the National Bureau of Standards as a possible secondary optical standard and also may be suitable as an interior coating for integrating spheres.

† F. Bickford of the Corning Research Laboratory directed efforts that led to paint development.

[1] Tomkins, M. and Tomkins, H., "The design of heat-reflective paints," J. Oil Colour Chemist Assoc. **41**, 98–108 (January 1958).

Measure of Satellite Dispersion

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A measure of symmetry is found for a set of coplanar vehicles. This measure can be applied to the case where the vehicles are launched simultaneously from a carrier vehicle. For a given period of time, an optimal burst configuration can be determined. However, as the period of time increases, the difference in the measure of an optimal and nonoptimal burst configuration approaches zero.

In the study of orbital configurations for satellites used in communications systems, it is highly desirable to have the vehicles in symmetric position. Unfortunately, because of launch considerations and perturbing influences, it is virtually impossible to maintain symmetric positioning. In particular, if a set of coplanar vehicles results from the subvehicles being launched simultaneously from a carrier vehicle, the configuration is subject to continual variation. Two problems arise. The first is to find some measure of symmetry, and the second is to determine optimal launch distribution of the subvehicles from the carrier vehicle. It is natural to take as a measure of symmetry the variations from the optimal symmetric state.

A necessary and subject condition that a symmetric distribution exists is that

$$\sum_{j=1}^{K} \binom{\sin}{\cos} M\alpha_j = 0$$

for a proper range of M, where α_j is the sum of the true anomaly and argument of perigee for the jth vehicle. This is because the symmetric distribution of K vehicles has an analogy in a display of the K roots of unity on the complex plane.

Proof: Given

$$\sum_{j=1}^{K} \binom{\sin}{\cos} M\alpha_j = 0 \qquad M = 1, 2, 3, \ldots$$

consider

$$\prod_{=1}^{K} (X - e^{i\alpha_j}) = 0$$

Then the expansion

$$X^K + a_1 X^{K-1} + a_2 X^{K-2} + \ldots + a_{K-1} X^1 + a_K = 0$$

has as coefficients

$$a_1 = -\sum_{j=1}^{K} e^{i\alpha_j} = 0$$

if

$$\sum_{j=1}^{K} e^{i\alpha_j} = 0$$

$$a_2 = + \sum_{\substack{1 \\ j_1 < j_2}}^{K} e^{i(\alpha_{j_1} + \alpha_{j_2})} = -\frac{1}{2} \sum_{j=1}^{K} e^{i2\alpha_j} = 0$$

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